# A Satisficing Fuzzy Logic Controller

M. A. Goodrich, W. C. Stirling , and R. L. Frost
Electrical and Computer Engineering Department
Brigham Young University,
Provo, UT 84604
wynn@ee.byu.edu

## Abstract

*Fuzzy logic controllers implicitly assume a utility maximizing principle. The satisficing principle is an alternative to utility maximizing. Instead of maximizing a single utility, the satisficing principle compares two independent utilities to determine admissible controls. Applying this principle to fuzzy logic controllers provides more defuzzifier latitude and, hence, can make the design of fuzzy logic controllers easier and more systematic.*

## 1 Introduction

As systems become more complex or less precisely known, the development of Conventional Controllers (CC's) becomes more difficult--a manifestation of Zadeh's *principle of incompatibility* [9]. Fuzzy Logic Controllers (FLC's) can be and have been successfully applied to many such systems, in part because of the simplified and tractable controller description. However, FLC's and CC's share a common foundation: each controller requires the generation and manipulation of a single utility. For FLC's, fuzzifiers and inference engines generate the utility of each control; for CC's, performance indices generate this utility. For FLC's, the defuzzifier uses this utility to produce a single ''best'' control; for CC's, the performance index is extremized to produce a single ''best'' control. Although the complexity is (perhaps greatly) reduced, FLC's implicitly attempt to maximize utility.

Utility maximization, however, is neither the only possible paradigm for rational decision-making, nor the appropriate paradigm for all decision-making scenarios. Simon [5] presents an alternative principle, termed the *satisficing principle*: all solutions that meet a minimum standard, possibly obtained under constraints of partial information or restricted computation, are admissible. Levi [3] provides an epistemological theory which is consistent with the satisficing principle. In contrast to the conventional practice of maximizing a single utility, Levi's theory requires that two independent utilities be compared. This structure forms the basis of the satisficing control theory presented in this paper.

We first present a theory of satisficing control based on Levi's epistemology. Based on this theory, we develop a satisficing fuzzy logic controller (SFLC). Finally, we demonstrate the use of an SFLC on the inverted pendulum problem.

## 2. Summary of Satisficing Control Theory

### 2.1. Satisficing Decisions

Control problems are usually specified in terms of two desiderata: (a) the ultimate goal of the controller (for example, to drive the state to a fixed set-point), and (b) the design principles used to generate a specific control policy (for example, a performance index to be minimized). To apply the satisficing principle, which selects controls by comparing these two desiderata, two utilities are required: one which reflects the ultimate goal of the controller and another which reflects the design principles. We call these two utilities the *accuracy* and *rejectability* utilities, respectively. We let these two utilities correspond to two fuzzy sets defined under the linguistic variable *value of control*. Let the set $U$ denote the control space $(u_{\min}, u_{\max})$ where we have assumed that the accuracy and rejectability utilities assign all of their mass to this interval; i.e., $U$ is the universe of discourse for the linguistic variable *value of control*.

We wish to define a utility function to characterize the accuracy, meaning *conformity to a given standard*, of a control $u \in U$. This utility has the form of a set membership function $\mu_A : U \mapsto [0,1]$. $\mu_A(u)$ is an accuracy-bearing utility function, termed the *accuracy* function. The accuracy may be interpreted as a measure of how well a control decision achieves the ultimate goal; that is, its effectiveness. This set membership function defined over $U$ defines the fuzzy set termed the *accuracy set*.

0-7803-3645-3/96 $5.00©1996 IEEE

Rejectability may be interpreted as a measure of how well a control decision obeys the design principles, independently of its effectiveness. The rejectability utility may be expressed through another membership function, $\mu_R : U \mapsto [0,1]$. $\mu_R(u)$ is the utility of rejecting the control $u \in U$. This set membership function defines the fuzzy set termed the *rejectability set.*

The *satisficing set* is defined as

$$S_b = \{u \in U : \mu_A(u) \geq b\mu_R(u)\}, \quad (1)$$

where $b$, the rejectivity index, establishes the satisficing threshold. We can, without loss of generality, restrict attention to rejectivity small enough to guarantee that $S_b$ defined in (1) is nonempty. In contrast to many decision-making procedures but in the spirit of fuzziness, this approach relaxes the requirement for a unique best decision and, instead, admits as satisficing all decisions for which accuracy exceeds rejectability. The satisficing set $S_b$ can be viewed as a fuzzy set with associated membership function

$$\mu_S(u;b) = \max\{0, \mu_A(u) - b\mu_R(u)\}. \quad (2)$$

Obviously, only one control from this fuzzy set can actually be implemented, but, from a strictly satisficing point of view, one may choose any of the unrejected control decisions with some confidence that the action will yield good, if not optimal, performance. Thus the designer has considerable latitude in the ultimate choice of the control to be implemented and, hence, in the selection of a defuzzification procedure. However, it is helpful to further restrict the satisficing set prior to defuzzifying.

## 2.2. Strongly Satisficing Control

Satisficing, as we have defined it, is a weak notion of performance: broadly speaking, a proposition is satisficing if the good (characterized by accuracy) outweighs the bad (characterized by rejectability). Furthermore, the satisficing set, $S_b$, will generally not be a singleton set and, hence, there may be many satisficing control possibilities. However, although all controls in $S_b$ are satisficing, they are not necessarily all equal. In selecting a control, if a choice exists between two controls of equal rejectability but differing accuracy, it is reasonable to prefer the one with higher accuracy. Similarly, if a choice exists between two controls of equal accuracy, it is reasonable to select the one with lower rejectability.

Three strongly satisficing controls are immediately obvious. A *most accurate* satisficing control, $u_A = \arg\max_{z \in S_b}\{\mu_A(z)\}$, would be appropriate for cases with large variations in $\mu_A$ relative to small variations in $\mu_R$. Such a decision represents a very aggressive stance to achieve the goal at the risk of excessive cost. A *least rejectable* satisficing control, $u_R = \arg\min_{z \in S_b}\{\mu_R(z)\}$, would be appropriate when there are large variations in $\mu_R$ relative to changes in $\mu_A$. This procedure is very conservative, and reflects a willingness to compromise the goal in the interest of reducing cost. A *most discriminating* satisficing control, $u_D = \arg\max_{z \in S_b}\{\mu_A(z) - b\mu_R(z)\}$ reflects a desire to compromise between cost and achievement in a way that maximizes the difference between the two.

It is desirable to identify the set, $\mathfrak{S}_b$, of all strongly satisficing solutions. Clearly, $\{u_A, u_R, u_D\} \subset \mathfrak{S}_b$. More generally, define

$$\begin{aligned} \mathfrak{S}_b \;=\; & \{u \in S_b : \nexists v \in S_b \text{ for which } \mu_R(v) < \mu_R(u) \\ & \text{and } \mu_A(v) > \mu_A(u)\}. \end{aligned} \quad (3)$$

The strongly satisficing set consists of all those satisficing controls for which no other satisficing control is both more accurate and less rejectable. We may interpret this by saying that the strongly satisficing set consists of controls for which no obviously better solution exists. In consequence of this, strongly satisficing solutions enjoy an equilibrium property: the accuracy cannot be increased without also increasing the rejectability, and the rejectability cannot be decreased without also decreasing the accuracy.

As with the satisficing set, the strongly satisficing set can be viewed as a fuzzy set with associated membership function

$$\mu_{\mathfrak{S}}(u;b) = \begin{cases} \mu_S(u;b) & u \in \mathfrak{S}_b \\ 0 & \text{otherwise} \end{cases}. \quad (4)$$

Controls in this fuzzy set have nonzero membership only when accuracy exceeds rejectability (times rejectivity) and when no other control is obviously better. A defuzzifier operating on this set has a great deal of latitude because each element in the set has a legitimate claim as the choice of control.

# 3. Fuzzy Logic and Satisficing Decisions

A conventional fuzzy logic controller is diagrammed in Figure 1. The output of the inference engine, when viewed as a function of potential controls, represents the utility of each control. In other words, the output of the inference engine defines a *solution fuzzy set* of controls with a corresponding degree of membership [1]. The defuzzifier manipulates this membership function to select a single control. Selection and design of a defuzzifier can be a significant obstacle in designing an FLC [4].

The SFLC diagrammed in Figure 2 includes both an accuracy utility as well as a rejectability utility. By comparing Figure 1 to 2, we see that the accuracy and rejectability set membership function pair in the SFLC can be used to refine the solution fuzzy set membership function prior to

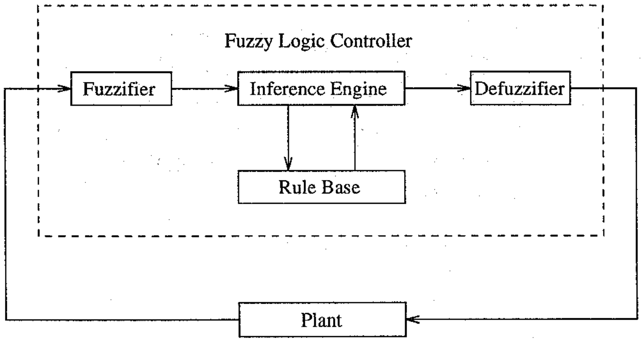


**Figure 1. Basic components of a conventional FLC.**

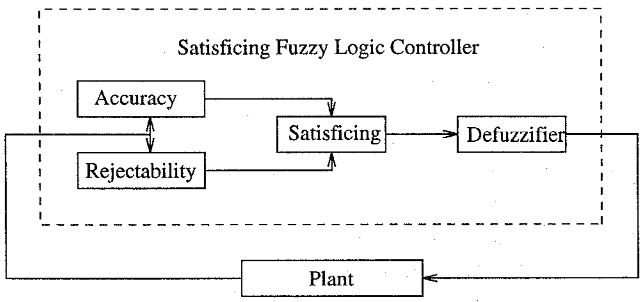


**Figure 2. Basic components of a satisficing FLC.**

defuzzifying. This should not be a large conceptual leap for practitioners of fuzzy logic. We have only added one additional membership function defined over the linguistic variable *value of control*, but by doing so we have potentially reduced the defuzzifier complexity since every strongly satisficing control is justifiable. Note that when $b = 0$ the conventional FLC is obtained and, thus, the SFLC can be viewed as a generalization of the FLC. In controlling the inverted pendulum, we demonstrate that the SFLC design is relatively simple, but produces very good results.

We wish to emphasize that the satisficing principle operates independently of how the accuracy and rejectability set membership functions are obtained; the satisficing principle does not depend on a particular fuzzification procedure or inference engine. It is for this reason that the accuracy and rejectability modules are represented as featureless boxes in Figure 2. A conventional rule-based system can be used to fuzzify inputs and infer a utility defined over possible controls, or cost functions can be used to generate the accuracy and rejectability membership functions. It is also possible for a combination of cost functions and rule bases to be used to generate these membership functions. In the next section, we present a fuzzification technique and inference engine that are implicitly incorporated in a cost-function formulation.

## 4. Control of the Inverted Pendulum

We now develop an SFLC for the inverted pendulum problem: *control an inverted pendulum in a vertical plane with full circular freedom by applying a lateral force to the cart to which the pendulum is attached, while* simultaneously *regulating the position of the cart to any desired point.* Traditional controllers must linearize the dynamics model of the pendulum in a small region within, say 10 deg of the vertical. More recently, an FLC trained by a genetic algorithm has been shown to balance the pendulum 90 % of the time if the pendulum is given a random initial position within 80 deg of the vertical and a random initial velocity less than 80 deg/sec [2]. We will design an SFLC for the given inverted pendulum problem given any initial cart position, cart velocity, pendulum angle, and pendulum angular velocity less than 80 deg per sample time $T$.

The inverted pendulum apparatus is illustrated in Figure 3, where $M$ is the mass of the cart, $l$ is the length of

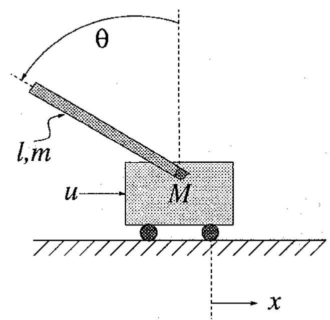


**Figure 3. Inverted pendulum on a cart.**

the pendulum, $m$ is the mass of the pendulum, $\theta$ is the angle from vertical (measured counterclockwise), $x$ is the horizontal position of the cart, and $u$, the control input, is a lateral force applied to the cart. The continuous-time dynamical equations for this problem are

$$\ddot{\theta} = \frac{(M+m)g\sin\theta - ml\cos\theta\sin\theta\dot{\theta}^2 - \cos\theta u}{l(M+m\sin^2\theta)} \tag{5}$$

$$\ddot{x} = \frac{ml\sin\theta\dot{\theta}^2 - mg\cos\theta\sin\theta + u}{M+m\sin^2\theta}. \tag{6}$$

Let $\mathbf{x}(k) = [\theta(k), x(k), \dot{\theta}(k), \dot{x}(k)]^T$ denote the state of the system. Using Euler integration, the equivalent discrete-time dynamical expression is $\mathbf{x}(k+1) = \mathbf{x}(k) + T\dot{\mathbf{x}}(k)$. With this discretization procedure, it is clear that $\theta(k+1)$ and $x(k+1)$ are not explicit functions of $u(k)$, but $\dot{\theta}(k+1)$, $\dot{x}(k+1)$, $\theta(k+2)$, and $x(k+2)$ are explicit functions of $u(k)$. Thus, we may identify the components of the state that are influenced by the current input $u(k)$ as the velocity vector $\mathbf{v}(k+1) = [\dot\theta(k+1), \dot x(k+1)]^T$ and position vector $\mathbf{p}(k+2) = [\theta(k+2), x(k+2)]^T$.

We adopt a quadratic performance index. Since the inverted pendulum is a regulator problem, the goal of the system is to bring the system to rest at the desired point. Let $Q_p$ and $Q_v$ be $2 \times 2$ cost matrices for position and velocity, respectively, and let $R$ be the cost matrix for control $u$. The accuracy cost functional is defined as

$$\Phi(u) = \mathbf{p}^T(k+2)\mathbf{p}(k+2) + \mathbf{v}^T(k+1)\mathbf{v}(k+1) \quad (7)$$

and the rejectability cost functional is defined as

$$\begin{aligned} \Lambda(u) &= \mathbf{p}^T(k+2)Q_p\mathbf{p}(k+2) + \\ &\quad \mathbf{v}^T(k+1)Q_v\mathbf{v}(k+1) + Ru^2. \end{aligned} \quad (8)$$

From these equations, $\mu_A$ and $\mu_R$ may be determined as $\mu_A(u) = \kappa_A\Big[\max_{z\in U}\{\Phi(z)\} - \Phi(u)\Big]$ and $\mu_R(u) = \kappa_R\Big[\Lambda(u) - \min_{z\in U}\{\Lambda(z)\}\Big]$ where $\kappa_A$ and $\kappa_R$ are the normalizing constants required to create (state-dependent) membership functions. Because any element in the strongly satisficing set can be justifiably applied as the control to the plant, the design of a defuzzifier can be simplified. One conventional defuzzifier determines the center of mass of the *solution fuzzy set* (as weighted by the utility function). For this construction, $\mu_{\mathfrak{S}}(u; b)$ is a concave function which means that the center of mass calculation is a reasonable defuzzifier. Elsewhere and in a different context, we have successfully applied even more simple defuzzifiers [7].

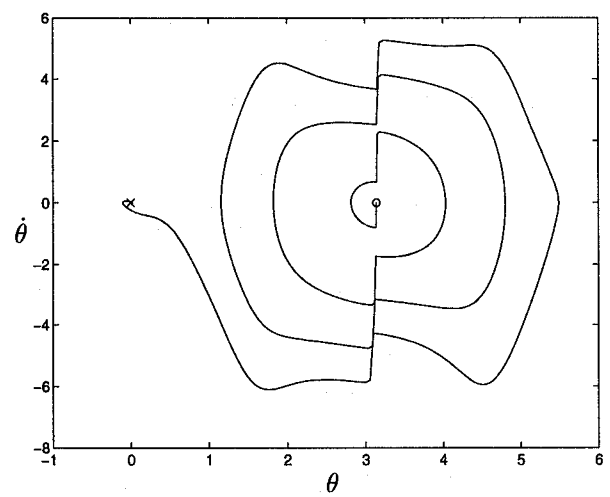


**Figure 4. Rotational phase planes for inverted pendulum on a cart (in radian per second and radians).**

The following values were used in the simulation: $Q_p(k) = \begin{bmatrix} 3 & 1 \\ 1 & 0.3 \end{bmatrix}$, $Q_v(k) = \begin{bmatrix} 0.2 & 0.2 \\ 0.2 & 0.2 \end{bmatrix}$, $R_u(k) = 10^{-4}, M = 0.455\,\text{kg}, m = 0.21\,\text{kg}, l = 0.61\,\text{m}, T = 0.01\,\text{s}$, and $U = [-1000, 1000]$. The off-diagonal terms in $Q_p$ and $Q_v$ reflect the coupling that must be modeled between $\theta$ and $x$ due to the fact that there is only one control input but two degrees of freedom in the system, namely the rotational and translational components.

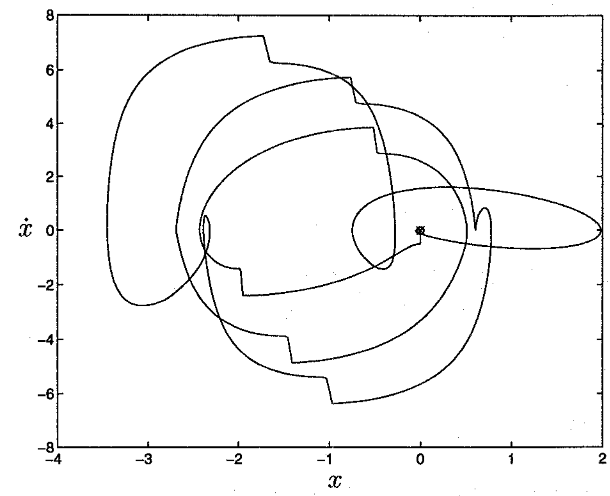


**Figure 5. Translational phase planes for inverted pendulum on a cart (in meters per second and meters).**

Figures 4 and 5 illustrate the rotational (pendulum) and translational (cart) phase planes. The "o" symbol represents the initial conditions (the cart at the origin with the pendulum in the vertical down position) and the "×" symbol represents the terminal conditions (the cart at the origin with the pendulum balanced in the vertical up position). The control history is shown in Figure 6. The system achieves its desired objective of balancing the pendulum at the origin by swinging the pendulum back and forth while the cart oscillates around the origin. As the cart oscillates, the pendulum gathers momentum. In the translational and rotational phase planes, this motion is manifest as growing spirals. When the amplitude increases sufficiently, the oscillation ceases and the pendulum then converges to the vertical upright position. Finally, the cart returns slowly to the origin.

## 5. Discussion

Fuzzy Logic Controllers usually determine a control by defuzzifying a single utility. This defuzzifying process implicitly assumes a utility maximizing perspective. Satisficing Fuzzy Logic Controllers use two utilities, and determine admissible controls by the interplay between these two utilities. The use of two utilities can make controller design more simple as well as create greater latitude in defuzzifier selection.

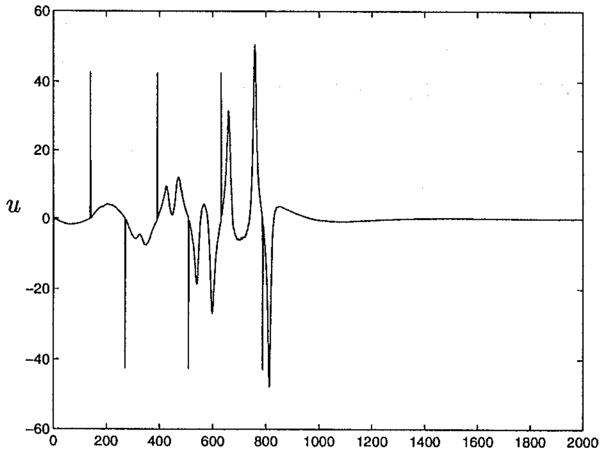


**Figure 6. Control history for inverted pendulum on a cart (in 0.01 sec increments).**

## References


[1] E. Cox. Fuzzy fundamentals. *IEEE Spectrum*, pages 58--61, October 1992.

[2] J. Kim, Y. Moon, and B. P. Zeigler. Designing Fuzzy Net Controllers Using Genetic Algorithms. *IEEE Control Systems Magazine*, 15(3):66--72, 1995.

[3] I. Levi. *The Enterprise of Knowledge.* MIT Press, Cambridge, Massachusetts, 1980.

[4] J. M. Mendel. Fuzzy logic systems for engineering: A tutorial. *Proceedings of the IEEE*, 83(3):345--377, March 1995.

[5] H. A. Simon. A Behavioral Model of Rational Choice. *Quart. J. Economics*, 59:99--118, 1955.

[6] W. C. Stirling. Coordinated Intelligent Control Via Epistemic Utility Theory . IEEE Control Systems Magazine, 13(5):21--29, October 1993.

[7] W. C. Stirling, M. A. Goodrich, and R. L. Frost. Procedurally rational decision-making and control. *IEEE Control Systems Magazine*, 1996. To Appear.

[8] W. C. Stirling and D. R. Morrell. Convex Bayes Decision Theory. *IEEE Transactions on Systems, Man, and Cybernetics*, 21(1):173--183, January/February 1991.

[9] L. A. Zadeh. The Concept of a Linguistic Variable and it Application to Approximate Reasoning--I. *Information Sciences*, 8:199--249, 1975.