A Probabilistic Bipartite Graph Model for Hub Based Swarm Solution of the Best-of-N Problem

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For *spatial swarms*, which are characterized by co-located agents, graph-based models complement agent-based and differential equation models, especially in providing theoretical results that apply to large-but-finite numbers of agents [4]. For *hub-based colony swarms*, where agents are often not in spatial proximity [3], except at a centralized hub, graph-based models do not appear to have received much attention. This extended abstract presents a graph-based model for a hub-based colony solving the best-of-N problem [5].

Hub-based colonies are characterized by two different kinds of entities: agents and sites (locations of interest in the world). Let $G = (V, E)$ be a bipartite graph with $V = V_{\text{agent}} \cup V_{\text{site}}$ partitioned into agent vertices and site vertices. Since G is bipartite, The edge set E has edges connecting an agent vertex only to a site vertex. A directed edge between agent a and a site s means the agent is "committed" to that site (assessing, promoting, committed to, etc.). The quality of sites is, without loss of generality, a real-valued number in $[0, 1]$.

Two probabilities determine the graph dynamics: the *attachment* probability, which determines when a new edge is formed between an agent and a site, and the *detachment* probability, which determines when an existing edge is removed. Attachment uses the preferential attachment pattern [1] and begins when an agent is randomly selected with uniform probability. If the agent is not connected, it randomly chooses a site to which it attaches with a probability proportional to the degree of the site. Detachment uses a tunable clustering pattern [2] and proceeds by selecting an edge with uniform probability from E. The probability that the edge is removed decreases linearly with site quality. Popularity-based clustering and degree-based persistence makes it likely that agents will cluster at the highest quality site, effectually solving the best-of-N problem. Figure 1 shows snapshots of an agent-based implementation of the graph dynamics.

Attachment and detachment induce graph dynamics for a discrete-time Markov process (DTMC) over a finite state space. MATLAB's `dtmc` method was used to compute a numerical solution for how the distribution evolves over time for nine agents and two sites. The quality of the best site was fixed to $\text{qual}(s_1) = 0.95$, and for the second best site was varied between $\text{qual}(s_0) \in \{0.05, 0.75\}$. Thus, the difference in qualities was $\Delta = \text{qual}(s_1) - \text{qual}(s_0) \in \{0.9, 0.2\}$.

Two initial distributions were considered: λ^{empty} placed all probability mass on the configuration with no edges, which represents a colony just beginning the best-of-N problem with no sites discovered. λ^{worst} placed all probability on the configuration with all agents connected to the second-best site, with its

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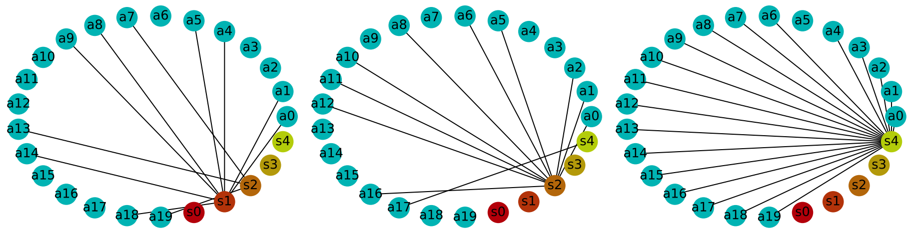

Fig. 1. Snapshots of graph configuration from agent-based simulation converging to highest quality site. From left to right $t = 50$, $t = 200$, $t = 350$.

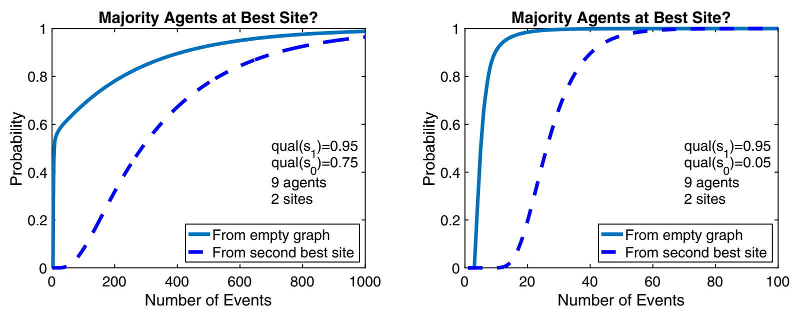

Fig. 2. Evolution of the DTMC state distribution from λ^{empty} (solid line) and from λ^{worst} (dashed line). Left: $\Delta = 0.20$, Right: $\Delta = 0.90$.

evolution representing the time taken by the colony to switch "commitment" from the inferior site to the superior site. Figure 2 shows the probability that a plurality of agents favors the superior site for a colony with 9 agents. Having similar site qualities slows convergence to the best site.

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