3. Given the query \( q = (k_1 \lor k_2) \land \neg k_3 \), and documents \( d_1, d_2, d_3, d_4, \) and \( d_5 \) in a text reference collection, and the following data:

<table>
<thead>
<tr>
<th>Index terms/documents</th>
<th>( d_1 )</th>
<th>( d_2 )</th>
<th>( d_3 )</th>
<th>( d_4 )</th>
<th>( d_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_1 )</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>( k_2 )</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>( k_3 )</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
<td>N</td>
</tr>
</tbody>
</table>

where Y (N, respectively) in the table denotes that the corresponding document contains (does not contain, respectively) the respective index term.

(a) [8 pts] Given that the normalized correlation factor is \( c(i, l) = n(i, l) / (n_l + n_i - n(i, l)) \), whereas the degree of membership of document \( d_j \) in the fuzzy subset associated with query term \( k_i \) is \( \mu(i, j) = 1 - \Pi_{k_i \in d_j} (1 - c(i, l)) \), calculate \( \mu(1, d_1), \mu(2, d_2), \) and \( \mu(3, d_1) \).

\[
\begin{align*}
\mu(1, d_1) &= \Pi (1 - 0.2) (1 - 0.2) = 0.56 \\
\mu(2, d_1) &= \Pi (1 - 0.25) (1 - 0.25) = 0.75 \\
\mu(3, d_1) &= \Pi 0.44 = 0.56
\end{align*}
\]

(b) [8 pts] Compute the similarity between \( q \) and \( d_1 \), given that \( \mu(q, d_1) = 1 - \Pi_{i=1, n} (1 - \mu(c_{c_i}, d_1)) \), where \( n \) is the number of conjunctive components of \( q \) and \( c_{c_i} \) is the \( i \)-th conjunct of \( q \).

Since \( (k_1 \lor k_2) \land \neg k_3 \) = \( (k_1 \land k_2) \lor \neg k_3 \) = \( (k_1 \land \neg k_2 \land k_3) \lor (k_1 \land k_2 \land k_3) \lor (k_1 \land \neg k_2 \land \neg k_3) \lor (k_1 \land k_2 \land \neg k_3) \lor (k_1 \land \neg k_2 \land \neg k_3) \) = \( (1, 1, 0) \lor (1, 0, 1) \lor (0, 1, 0) \lor (1, 1, 1) \) 

Hence, \( \mu(q, d_1) = 1 - (1 - \mu(1, d_1) \mu(2, d_1) \mu(3, d_1)) \times (1 - \mu(1, d_1) \mu(2, d_1) \mu(3, d_1)) \times (1 - \mu(1, d_1) \mu(2, d_1) \mu(3, d_1)) \times (1 - \mu(1, d_1) \mu(2, d_1) \mu(3, d_1)) \times (1 - \mu(1, d_1) \mu(2, d_1) \mu(3, d_1)) \)

\[
= 1 - (1 - 0.56) (1 - 0.56) (1 - 0.56) = 1 - 0.56 
\]

\[
0.56
\]