

Chapter 10

The Chomsky Hierarchy

Grammars Languages & Accepting Machines

Grammars	Languages	Accepting Machines
Type 3 grammars: Regular grammars, Left-linear grammars, Right-linear grammars	Regular languages	DFA N DFA
Type 2 grammars: Context-free grammars	Context-free languages	PDA
Type 1 grammars: Context-sensitive grammars, Monotonic grammars	Context-sensitive languages	Linear-bounded Automata
Type 0 grammars: Phrase-structure grammars, Unrestricted grammars	Recursively enumerable, Unrestricted	DTM ND TM

Chapter 10. Unrestricted Grammars

- The highest class of the *Chomsky hierarchy* and the largest class (i.e., most powerful) of *phrase-structured grammars*, called **type 0** grammars.
- Defn.10.1.1 An unrestricted grammar is a quadruple (V, Σ, P, S) , where
 - V : a finite set of *variables* (i.e., *non-terminal* symbols)
 - Σ : the *alphabet*, a finite set of *terminal* symbols
 - P : a set of production rules of the form $u \rightarrow v$,
where $u \in (V \cup \Sigma)^+$ and $v \in (V \cup \Sigma)^*$, i.e., $u \neq \lambda$
 - S : the start symbol

Unrestricted Grammars

■ Example 10.1.1 The unrestricted grammar

$$V = \{ S, A, C \}$$

$$\Sigma = \{ a, b, c \}$$

$$P = \{ S \rightarrow aAbc \mid \lambda$$

$$A \rightarrow aAbC \mid \lambda$$

$$Cb \rightarrow bC$$

$$Cc \rightarrow cc \}$$

generates the language $\{ a^i b^i c^i \mid i \geq 0 \}$ (which is not context-free) as

$$S \Rightarrow aAbc \xRightarrow{i-1} a^i A (bC)^{i-1} bc \Rightarrow a^i (bC)^{i-1} bc$$

$$\Rightarrow a^i (bC)^{i-2} bCbc$$

$$\Rightarrow a^i (bC)^{i-2} bbCc$$

$$\xRightarrow{*} a^i (b)^i (C)^{i-1} c$$

$$\xRightarrow{i-1} a^i b^i c^i$$