Discussion #31

Adjacency Lists;
Breadth-First Search &
Depth-First Search
Topics

• Adjacency list representation
• Breadth-First Search (BFS)
• Depth-First Search (DFS)
Adjacency Lists

For directed graphs:

Simple Notation

\{ (a,a), (a,b), (a,c), (b,c), (c,b) \}
Adjacency Lists for Undirected and Weighted Graphs

- **Undirected Graphs:**
  - Make each edge (except loops) go both ways.
  
- **Weighted Graphs:**
  - Add additional field to node
  - Node-weight pairs

### Undirected Graphs

- **Diagram:**
- **Table:**
<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>b</td>
<td>a</td>
<td>c</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>a</td>
<td>b</td>
<td></td>
</tr>
</tbody>
</table>

### Weighted Graphs

- **Table:**
<table>
<thead>
<tr>
<th></th>
<th>(a,1)</th>
<th>(b,5)</th>
<th>(c,3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>(a,5)</td>
<td>(c,7)</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>(a,3)</td>
<td>(b,7)</td>
<td></td>
</tr>
</tbody>
</table>
Breadth-First Search (BFS)
BFS Algorithm

Undirected edges: each edge twice

O(n) = O(n+2m) = O(m) if m >> n

~ 2m edges

queue: a b c d
Depth-First Search (DFS)
DFS Algorithm

- Nodes: \( n \) nodes
- Stack: a b c d
- Time complexity:
  - \( \text{nodes} \): \( O(n) \)
  - \( \text{edges} \): \( \sim 2m \)

### Algorithm Steps

1. Start with node a
2. Explore connected nodes:
   - b → a
   - c → a, b
   - d → b, c
3. Mark visited nodes: a, b, c, d

### Time Complexity

- \( O(n) \) for nodes
- \( O(m) \) for edges

### Complexity Analysis

- If \( m \gg n \), then:
  - \( O(n+2m) \)
  - \( O(m) \)

### Example Graph

![Graph Diagram](image)
BFS vs. DFS

BFS Queue

a b c e d f

Done!

BFS Order: a b c e d f
BFS vs. DFS

**BFS Queue**

\[
\text{Done!}
\]

**BFS Order:** \(a \ b \ c \ e \ d \ f\)

**DFS Stack**

\[
\text{Done!}
\]

**DFS Order:** \(a \ b \ c \ f \ d \ e\)

Discussion #31
BFS & DFS with Directed Graphs

BFS

a,b,c,d,e,f,g,h,i,j,k,l,m,n

Same as before, by chance

DFS

a,b,e,j,f,k,l,h,c,g,d,i,m,n

Not same as before