Winter 2001
Computer Aided Geometric Design
Due Monday, 16 April 2001

1. $a = [-2, 3]$ and $b = [3, 4]$. What is $a * b$?

What is b^2?

What is a^3?

What is a/b?

What is b/a?

What is 2^a?

What is $a^2 + 2a + 3$?

2. A Bernstein polynomial with interval coefficients is given as

$$p(t) = [1, 3] * (1 - t)^2 + [3, 5] * 2t(1 - t) + [2, 4]t^2.$$

Use the deCasteljau algorithm to compute:

$p(\frac{1}{2}) =$

$p(2) =$

$p(4) =$

3. For a certain degree two polynomnial, $p(1) = [.999, 1.001]$, $p(2) = (1.999, 2.001)$, $p(3) = (3.999, 4.001)$. Use forward differencing to compute $p(4)$ and $p(5)$.

4. For a certain degree two polynomnial, $p(1) = [.999, 1.001]$, $p(2) = (1.999, 2.001)$, $p(4) = (3.999, 4.001)$. Use Newton polynomials to compute $p(3)$.

5. Interval arithmetic can be used to robustly find all roots of an equation $p(t) = 0$. A simple way to do this is to use interval halving. If $0 \notin p([a, b])$, then there is no root for $t \in [a, b]$. If $0 \in p([a, b])$, check to see if $0 \in p([a, \frac{a+b}{2}])$ and $0 \in p([\frac{a+b}{2}, b])$. Use the interval halving method to find an interval whose width is $\frac{1}{2}$ that contains a root of $p(t) = 12t^2 - 31t + 20$. Start with the interval $[0, 4]$.

6. Use Bézier clipping to find values of t for which the curve cubic Bézier curve $\mathbf{P}(t)$ does not intersect thedegree one Bézier curve $\mathbf{Q}(s)$.
The control points of $\mathbf{P}(t)$ are:

$$\mathbf{P}_0 = (3, 0); \quad \mathbf{P}_1 = (1, 0); \quad \mathbf{P}_2 = (1, 1); \quad \mathbf{P}_3 = (3, 3);$$

The control points of $\mathbf{Q}(s)$ are

$$\mathbf{Q}_0 = (0, 0); \quad \mathbf{Q}_1 = (4, 2).$$