

CS 557
Homework 8
Due Friday, 6 April 2001

1. Parametrize the curve

$$x^2 + y^2 - 4x + 6y = 0.$$

2. Find the double points on the curve

$$1 + 4x + 6x^2 + 4x^3 + x^4 + 4y + 12xy + 12x^2y + 4x^3y + 6y^2 + 12xy^2 + 6x^2y^2 + 4y^3 + 4xy^3 + y^4$$

You may use the Grobner basis applet if you like.

3. Find the coordinates of the double point on the curve

$$x^3 + y^3 + xy - 5y^2 - 5x + 7y = 0.$$

4. Parametrize the curve in problem 3.

5. How many double points must a degree 5 curve have in order to be rational?

6. A biquadratic surface patch is given by the equation

$$\mathbf{P}(s, t) = \sum_{i=0}^2 \sum_{j=0}^2 \mathbf{P}_{ij} B_i^2(s) B_j^2(t).$$

If

$$\mathbf{P}_{00} = (0, 0, 0) \quad \mathbf{P}_{10} = (2, 0, 2) \quad \mathbf{P}_{20} = (4, 0, 0)$$

$$\mathbf{P}_{01} = (0, 2, 2) \quad \mathbf{P}_{11} = (2, 2, 4) \quad \mathbf{P}_{21} = (4, 2, 2)$$

$$\mathbf{P}_{02} = (0, 4, 2) \quad \mathbf{P}_{12} = (2, 4, 0) \quad \mathbf{P}_{22} = (4, 4, 2)$$

Find $\mathbf{P}(\frac{1}{2}, \frac{1}{2})$.

Find $\mathbf{P}_s(\frac{1}{2}, \frac{1}{2})$ and $\mathbf{P}_t(\frac{1}{2}, \frac{1}{2})$.

Find the normal to the surface at $\mathbf{P}(\frac{1}{2}, \frac{1}{2})$.