CS 557 W01
Homework #5
Due Friday, 9 February 2001

1. The rational Howard curve curve is given by

$$\mathbf{P}(t) = \frac{2-2t}{2-t}\mathbf{P}_0 + \frac{3t(t-1)}{(1+t)(2-t)}\mathbf{P}_1 + \frac{2t}{1+t}\mathbf{P}_2$$

a. Does this curve interpolate the endpoints? Why or why not?
b. Is this curve symmetric? Why or why not?
c. Is this curve coordinate system independent? Why or why not?
d. Does this curve obey the convex hull property? Why or why not?

2. Recall that the equation for Ball's cubic curve is:

$$(1-t)^2\mathbf{Q}_0 + 2t(1-t)^2\mathbf{Q}_1 + 2t^2(1-t)\mathbf{Q}_2 + t^2\mathbf{Q}_3.$$

Given a Ball curve with control points $\mathbf{Q}_0$, $\mathbf{Q}_1$, $\mathbf{Q}_2$, $\mathbf{Q}_3$, find the control points $\mathbf{P}_0$, $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$ of an equivalent cubic Bézier curve. (Hint: write down what you know about the relationship between Ball curves and Bézier curves.)

3. The *weight ratio* of a rational Bézier curve is the largest divided by the smallest weight. For example, the weight ratio of a cubic Bézier curve with weights $8, 2, 3, 4$ is 4. with Given a rational cubic Bézier curve with weights 1,4,12,27, reparameterize the curve so that the weight ratio is less than 2.

4. A quadratic B-spline has polar values whose Cartesian coordinates are

$$f(-1,0) = (4,4); \quad f(0,1) = (1,2); \quad f(1,2) = (2,0).$$

What are the control points of the equivalent Bezier curve?

5. A degree four B-spline has a knot vector [a b c d e f g h i j k l m n o p q r s]. None of the knots are multiple. If the control point $f(cdef)$ is moved, what are the parameter ranges of the underlying Bézier curves that are changed?

6. A degree n B-spline with m control points in general position has

________ curve segments,

________ knot values in the knot vector

________ order continuity at single knots.

7. Given two Bézier curves, one with control points

$$\mathbf{P}_0 = (0,0), \quad \mathbf{P}_1 = (1,2), \quad \mathbf{P}_2 = (2,2), \quad \mathbf{P}_3 = (3,2)$$

and the other with control points

$$\mathbf{Q}_0 = (3,2), \quad \mathbf{Q}_1 = (4,2), \quad \mathbf{Q}_2 = (5,2), \quad \mathbf{Q}_3 = (6,4)$$

express these two Bézier curves in B-spline form using as few B-spline control points and knots as possible.