

Winter 2001
Computer Aided Geometric Design
Homework #2
Due Friday, 19 January 2001

1. Given a rational cubic Bézier curve with control points and weights

$$\mathbf{P}_0 = (0, 0, w = 1) \quad \mathbf{P}_1 = (2, 0, w = 2) \quad \mathbf{P}_2 = (0, 4, w = 8) \quad \mathbf{P}_3 = (8, 8, w = 4)$$

subdivide at $t = \frac{1}{2}$ and find the control points and weights for the left half of the curve.

2. Degree elevate the curve in problem 1.

3. For a degree n polynomial Bézier curve whose first three control points are

$$\mathbf{P}_0 = (0, 0) \quad \mathbf{P}_1 = (a, 0) \quad \mathbf{P}_2 = (x_2, h)$$

derive the equations for the first derivative vector $\mathbf{P}'(0)$ and for the curvature $\kappa(0)$ for a polynomial Bézier curve (see pages 24 and 26 of the notes).

4. Find the control points and weights, letting $w_0 = w_2 = 1$, for a rational quadratic Bézier curve which exactly represents a quarter circle:

$$\mathbf{P}_0 = (1, 0, w = 1) \quad \mathbf{P}_1 = (1, 1, w = w_1) \quad \mathbf{P}_2 = (0, 1, w = 1)$$

Do it a second time letting $w_0 = 1$ and finding w_1 and w_2 as integers. Use Bezout's law in your derivation. Check your answer with the formula on page 27 of the notes.

5. Find the control points and weights for a rational cubic Bézier curve which exactly represents a semicircle (Let $w_0 = w_3 = 1$; $y_1 = y_2$; and $w_1 = w_2$):

$$\mathbf{P}_0 = (1, 0, w = 1) \quad \mathbf{P}_1 = (1, y_1, w = w_1) \quad \mathbf{P}_2 = (-1, y_2, w = w_1) \quad \mathbf{P}_3 = (-1, 0, w = 1)$$

6. Find $y_1 = x_2$ for which the following polynomial cubic Bézier curve most nearly approximates a quarter circle. What is the maximum error? (Use trial and error).

$$\mathbf{P}_0 = (1, 0) \quad \mathbf{P}_1 = (1, y_1) \quad \mathbf{P}_2 = (x_2, 1) \quad \mathbf{P}_3 = (0, 1)$$