Name: _____

150 points
Thursday, 17 December 1998 7-10 A.M.

Three pages of notes allowed. Show all your work.

1. (10 points) Circle TRUE or FALSE for each statement:

T **F** The Lagrange basis obeys the variation diminishing property.

T **F** The genus of a degree n Bézier curve is $(n-1)(n-2)/2$.

T **F** Any Bézier curve can be represented as a single B-spline curve.

T **F** Any B-spline can be represented as a single Bézier curve.

T **F** The "R" in NURBS stands for "rectangular".

2. (20 points) The following questions deal with the execution time of algorithms and are multiple choice. If for a degree $n = 100$ curve, the algorithm takes k units of time to execute, approximately how long would the algorithm take on a degree $n = 200$ curve? Possible answers are:

a. $2k$ b. $3k$ c. $4k$ d. k^2.

____ **a.** The de Casteljau algorithm.

____ **b.** The algorithm for converting the parametric equations of a Bézier curve to power basis.

____ **c.** Given $n+1$ points $\mathbf{P}(0)$, $\mathbf{P}(\delta)$, $\mathbf{P}(2\delta)$, ..., $\mathbf{P}(n\delta)$, on a curve known to be a degree n polynomial Bézier curve (but you don't know the control points) find $\mathbf{P}((n+1)\delta)$ using forward differencing.

____ **d.** The algorithm for evaluating a point on a degree n Lagrange curve.

____ **e.** Neville's algorithm for a degree n curve.

____ **f.** Horner's algorithm for a degree n curve in power basis.

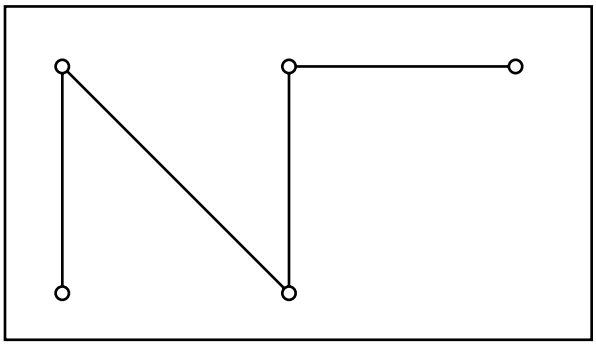
____ **g.** Degree elevation of a degree n Bézier curve.

____ **h.** An algorithm to find the minimum number of line segments (with evenly spaced parameter values) needed to plot a degree n Bézier curve so that the error is less than a specified value.

____ **i.** Böhm's algorithm for a degree n B-spline.

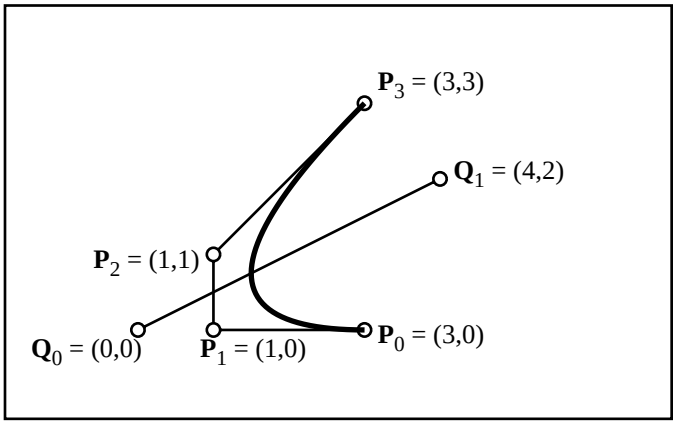
____ **j.** de Boor's algorithm for a degree n B-spline.

3. (6 points) Sketch the Overhauser curves defined by these control points. Pay attention to tangent directions.



4. (10 points) Write the equation of a curve $\mathbf{P}(t)$ which interpolates the points $\mathbf{P}_1$, $\mathbf{P}_2$, and $\mathbf{P}_3$ at parameter values 0, 1, and 3 respectively.

5. (10 points) Use Bézier clipping to find values of t for which the curve $\mathbf{P}(t)$ does not intersect the curve $\mathbf{Q}(s)$.



6. (6 points) Find the values of slope m for which the line

$$x = t; \quad y = mt$$

intersects the curve

$$2x^2y - 3xy^2 + y^3 + x^2 - 2xy + y^2 - 2y + 2x = 0$$

three times at the origin. (Note: Because this happens, this curve has an inflection point at the origin.)

Answer: $m =$

7. (10 points) Consider the parametric curve

$$\mathbf{P}(t) = \mathbf{P}_0 B_0(t) + \mathbf{P}_1 B_1(t) + \mathbf{P}_2 B_2(t)$$

where

$$\begin{aligned} B_0(t) &= 1 - 5t + 10t^2 - 10t^3 + 5t^4 - t^5 = (1-t)^5 \\ B_1(t) &= 5t - 10t^2 + 10t^3 - 5t^4 = 5t(1-t)(1-t+t^2) \\ B_2(t) &= t^5 \end{aligned}$$

a. YES NO Is the curve coordinate system independent? Why or why not?

b. YES NO Does the curve obey the convex hull property? Why or why not?

c. YES NO Is this curve symmetric? Write the equations that must be satisfied for symmetry.

d. YES NO Does the curve interpolate $\mathbf{P}_0$ and $\mathbf{P}_2$?

e. Compute $\mathbf{P}'(0)$.

8. (6 points) What are the homogeneous coordinates of all points (X, Y, W) at which the curve

$$(2X - 3Y)(X + Y) + (5X - 2Y)W + 4W^2 = 0$$

intersects the line at infinity? What kind of a curve is this? (Ellipse, parabola, hyperbola)

9. (6 points) Find the resultant of

$$f(t) = (t-1)(t-2)(t+3)$$

$$g(t) = t(t+2)(t+3)$$

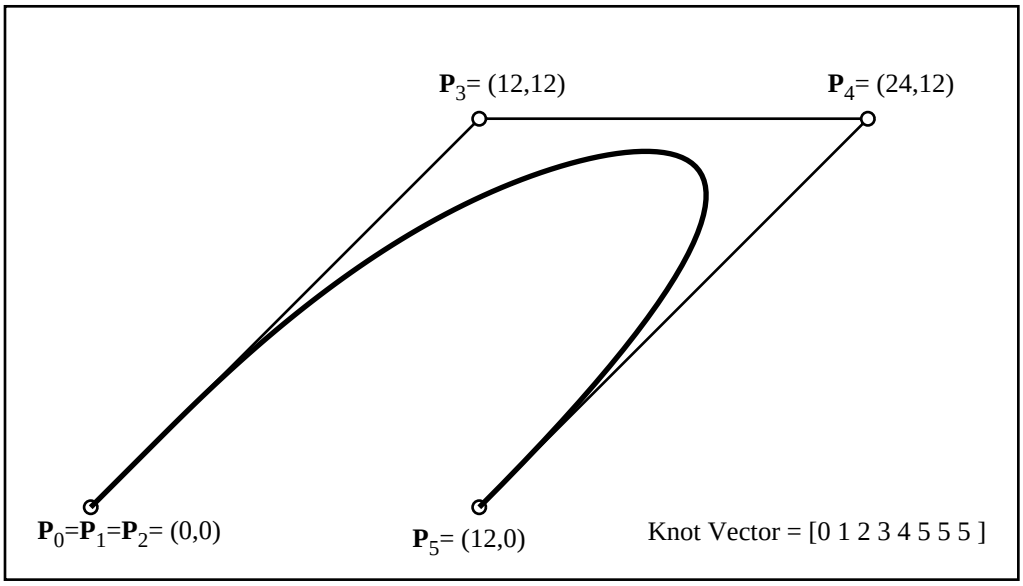
10. (20 points) Sometimes, people create a Bézier-like end condition on a degree n B-spline curve by placing n control points on top of each other, rather than using multiple knots. In the B-spline shown,

a. How many distinct Bézier curves are there?

b. What are the parameter limits for each of those Bézier curves?

c. What are the control points for each of those Bézier curves?

d. What order of continuity is there between each pair of Bézier curves?



11. (10 points) A certain *degree three* curve $f(x, y) = 0$ is intersected with a rational degree one Bézier curve with control points and weights

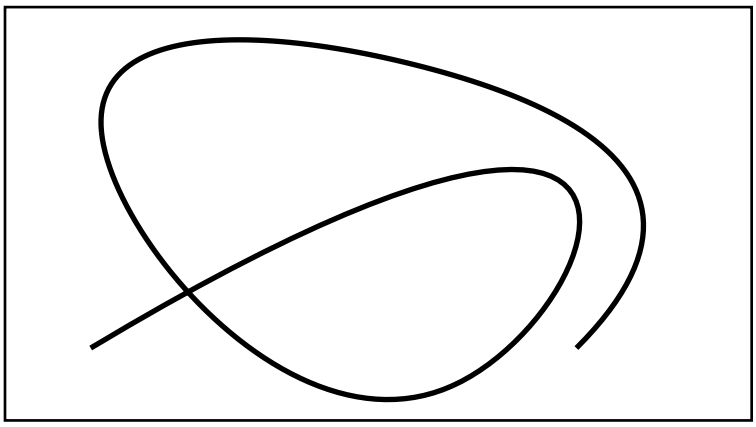
$$\mathbf{P}_0 = (0, 0);\ w_0 = 1; \qquad \mathbf{P}_1 = (3, 2);\ w_1 = 2$$

by substituting the parametric equation of the Bézier curve into the implicit equation of the other curve. This procedure produces a polynomial

$$t^2 - 2t = 0.$$

What are the (x, y) coordinates of all the points at which these two curves intersect?

12. (5 points) Indicate at which points the following curve must be subdivided as a preprocess for the Mudur-Koparkar interval algorithm for curve intersection.



13. (10 points) If the curve $f(x, y) = 0$ intersects the curve $g(x, y) = 0$ at points $(0, 2)$, $(3, 3)$, $(1, 2)$, and $(0, 0)$, what is the resultant of f and g with respect to y? (Both curves are degree two).

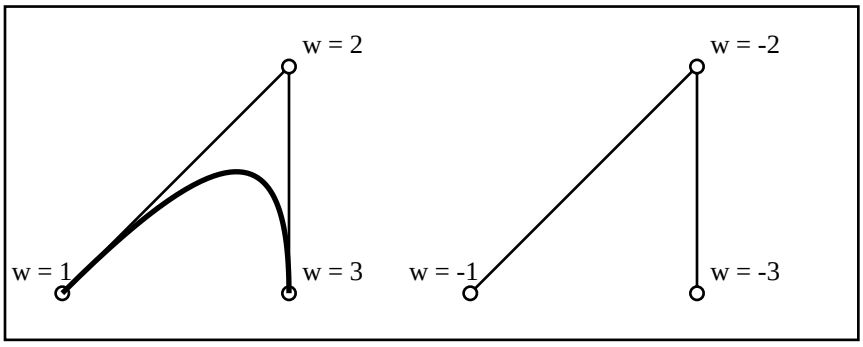
14. (6 points)

$$f(t) = t^5 - 2t^4 + t^3 - t^2 + 2t - 1$$
$$g(t) = t^4 - 2t^3 + 2t - 1$$

Find the GCD of $f(t)$ and $g(t)$. Find also the common roots of $f(t)$ and $g(t)$.

GCD = _____ Common roots = _____

15. (5 points) If the weights of this rational Bézier curve are all negated, sketch what the curve would look like.



16. (10 points) Find an inversion equation for the curve

$$x = \frac{t^2 + 2t + 2}{t^2 + 1}; \quad y = \frac{2t^2 - t + 1}{t^2 + 1}.$$

Use it to find the parameter value at the point $(1, 2)$.

Equation: _____ t at $(1, 2)$: _____