Artificial Intelligence

CS 470-1

Uncertainty and Utility Theory Homework

Homework Motivation. Now that you've had some experience with probabilistic reasoning and understand the basic concept of a utility, we can start exploring how to combine utility theory and probability theory together to make a decision. This set of homework has you make a decision using probability theory only, and then evaluate how you refine your decision when you consider not only probability but also utility. I also want you to explore how utility functions can be constructed to satisfy the needs of a problem.

Goal. Understand how to construct utilities for use in interesting problems, use expected utility theory in interesting problems, and how expected utility theory differs from probability theory alone.

1. Consider the problem of detecting an incoming missile given an alarm. The alarm is either on (corresponding to percept $o = 1$) or off (corresponding to percept $o = 0$). The world is such that there is either a missile (corresponding to state $s = 1$) or no missile (corresponding to state $s = 0$). You know the following probabilities:

$P(o = 0\|s = 0) = 0.9$	$P(o = 0\|s = 1) = 0.2$
$P(o = 1\|s = 0) = 0.1$	$P(o = 1\|s = 1) = 0.8$
$P(s = 0) = 0.7$	$P(s = 1) = 0.3$

An alarm occurs. What is the most probable explanation? (To get full credit, you must show your work.)

2. Consider a problem similar to the missile detection problem above. Suppose that you make an observation $o = 1$ (the alarm occurs) and determine that the probability of an incoming missile given the observation is $P(s = 1|o = 1) = 0.01$, and the probability of a false alarm given the observation is $P(s = 0|o = 1) = 0.99$. You have two actions that you can take: a_{launch} is the option of launching a defensive weapon to defuse the missile, and a_{wait} is the option of doing nothing. You have constructed the following utilities: $\mathcal{U}(a_{\text{wait}}; s = 0) = 0.5$, $\mathcal{U}(a_{\text{launch}}; s = 0) = 0.4$, $\mathcal{U}(a_{\text{wait}}; s = 1) = 0.0$, and $\mathcal{U}(a_{\text{launch}}; s = 1) = 8.0$. For example, $\mathcal{U}(a_{\text{wait}}; s = 0)$ is the utility of doing nothing when no missile is coming in. What is the optimal solution? (Hint, you must show how you derive the utility of both options to get credit.)

3. Suppose that you are choosing between four alternatives that are ranked $A \succ B \succ C \succ D$, and that you want to construct the utility function to represent your strength of preference. Suppose further that you set $U(A) = 100$ and $U(D) = 10$. Finally, suppose that you identify $p = 0.8$ and $q = 0.3$ as the values for which $B \sim [A, p; C, 1 - p]$ and $C \sim [B, q; D, 1 - q]$. What is $U(B)$ and $U(C)$?

4. The following table represents a multi-attribute problem where $\mathcal{U}_i$ denotes the i^{th} utility function and where c_j denotes a possible choice. Which choices are not dominated?

Utility/action	c_1	c_2	c_3	c_4	c_5	c_6
$\mathcal{U}_1$	3.5	2	3	4	5	2
$\mathcal{U}_2$	0	-1	-3	-4	-5	1
$\mathcal{U}_3$	1	6	0	4	2	2